Test No. 2

(1a) Use the first–scattered distributed source technique together with diffusion theory to solve for the scalar flux in a slab of width x_0, with an isotropic boundary flux, $\frac{\phi_0}{2\pi}$ $p/(cm^2 - sec - steradian)$, incident at the left face ($x = 0$) and a vacuum boundary condition at the right face ($x = x_0$). The slab is a pure scatterer, i.e., $\sigma_t = \sigma_s$, and the cross–section is constant in space. Note that to get the particular solution for this problem, you must use the fact that $\frac{d}{dx}E_n(x) = -E_{n-1}(x)$.

(1b) Calculate the fraction of particles entering the slab that are reflected.

(1c) Evaluate (1a) and (1b) in the limit as $x_0 \to \infty$

(2a) Use diffusion theory to calculate the scalar flux in a semi–infinite slab with an isotropic boundary flux, $\frac{\phi_o}{2\pi}$ $p/(cm^2 - sec - steradian)$, incident from the left. The cross sections are constant in space and there is both absorption and scattering: $\sigma_t = \sigma_a + \sigma_s$. The boundary condition at infinity is $\phi(\infty) < \infty$.

(2b) Calculate the fraction of particles entering the slab that are reflected.

(2c) Evaluate (2a) and (2b) in the limit as $\sigma_a \to 0$.

(2d) Evaluate (2b) with $\sigma_s = 0$ in the limit as $\sigma_a \to \infty$.

(3a) Solve the following problem analytically:

$$\frac{d\psi}{dx} + \sigma_a \psi = 0\,, \quad \text{for } x \in [0, x_0], \quad \text{with } \; \psi(0) = 1.$$

(3b) Solve this equation using a Petrov-Galerkin approximation with the following trial space:

$$\begin{aligned} \tilde{\psi}(x) &= 1.0 \quad \text{at } x = 0\,, \\ &= a + bx\,, \quad \text{otherwise,} \end{aligned}$$

and the following weighting space:

$$\begin{aligned} W_1(x) &= 1.0\,, \\ W_2(x) &= \delta(x - 2x_0/3)\,. \end{aligned}$$

(3c) Determine the order accuracy of the numerical solution for $\psi(x_0)$, i.e., determine n where

$$\psi(x_0)^{\texttt{EXACT}} = \psi(x_0)^{\texttt{NUMERICAL}} + O(x_0^n).$$

* Remember that the derivative of a discontinuous function of x is given by the change in the function in the direction of increasing x at the discontinuity times a delta-function defined at the point of discontinuity:

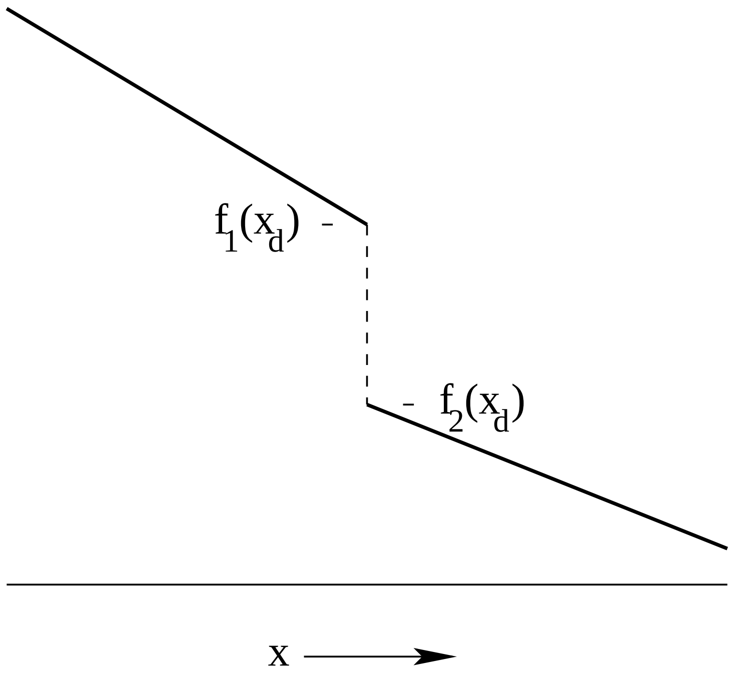

Figure 1: A discontinuous function.

$$\frac{df}{dx}|_{x=x_d} = (f_2 - f_1)\delta(x - x_d)$$

* Also remember that the need for representing the derivative at the point of discontinuity can be avoided simply by integrating the derivative term by parts.